

Exercises

Fatigue Analysis Course at TMC (13 & 27 september 2019)

Exercise

- Circular hole in biaxial stress field

- $\gamma=0.5$
- Calculate K_t at A

- Solution

- "Vertical" stresses

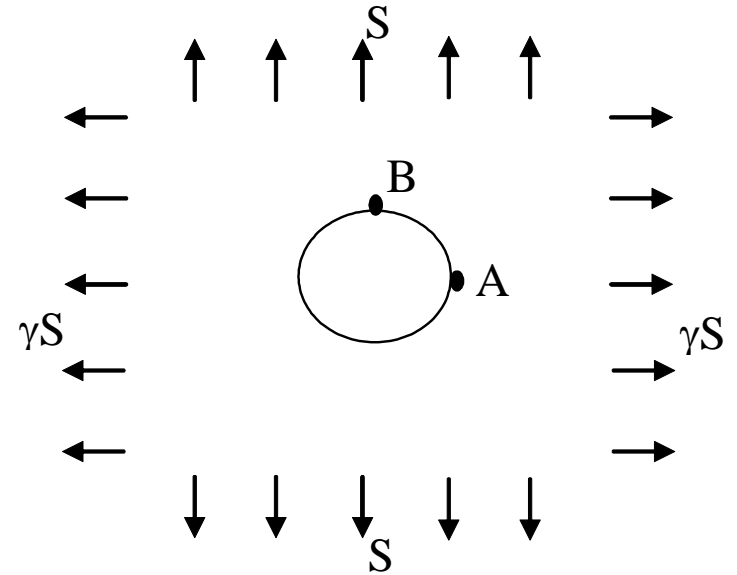
$$\sigma_A = 3S \quad \sigma_B = -S$$

- "Horizontal" stresses

$$\sigma_B = 3\gamma S = 1.5S \quad \sigma_A = -\gamma S = -0.5S$$

- Location A

$$\sigma_A = 3S - 0.5S = 2.5S \quad \Rightarrow \quad K_t = 2.5$$



Exercise

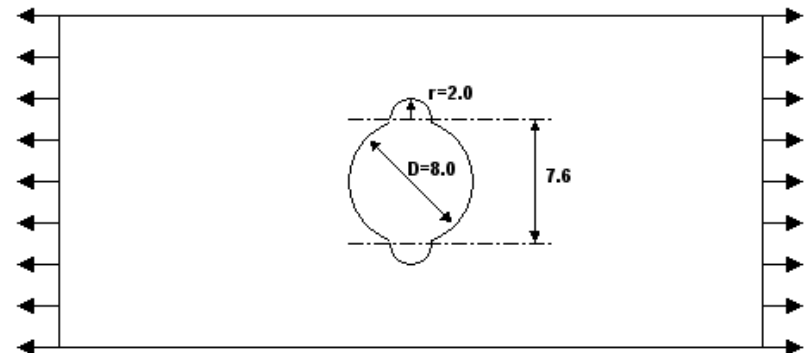
- a) What is the stress concentration factor of a circular hole ($D = 11.6$ mm) in an infinite sheet?
- $K_t = 3$
- b) Make an estimation of the K_t value of this notch for the case the notch is in a finite sheet with width of 40mm. Use the expression for the K_t in a finite width sheet with a circular hole according to Heywood:

$$K_t = 2 + \left(1 - \frac{d}{W}\right)^3 = 2 + \left(1 - \frac{11.6}{40}\right)^3 = 2.4$$

- c) Consider a specimen with notch in an infinite sheet as in the figure below. What is the stress concentration factor for that notch?

$$K_t = 1 + 2\sqrt{\frac{a}{\rho}}$$

$$K_t = 1 + 2\sqrt{\frac{5.8}{2}} = 4.4$$

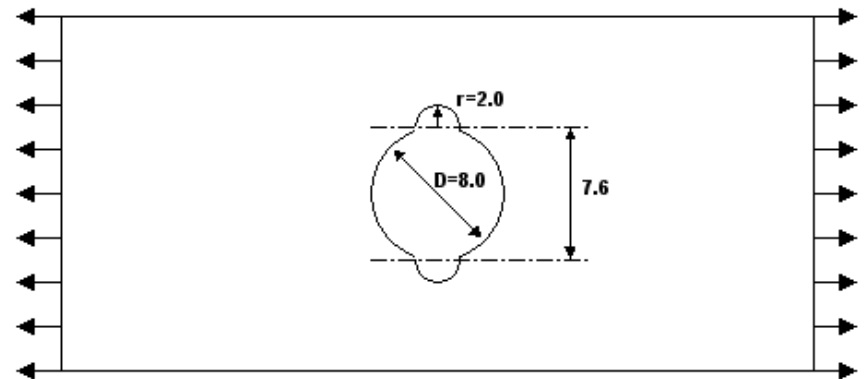


Exercise

d) Now assume a sheet width of 40 mm. Estimate the K_t value of the notch in the figure below based upon the previous questions

- Estimation using:
 - width effect for circular hole
 - notch, infinite sheet

$$K_t = \frac{2.36}{3} \cdot 4.41 = 3.46$$



e) Calculate the theoretical (elastic) value of the maximum stress at the notch for case d), with:

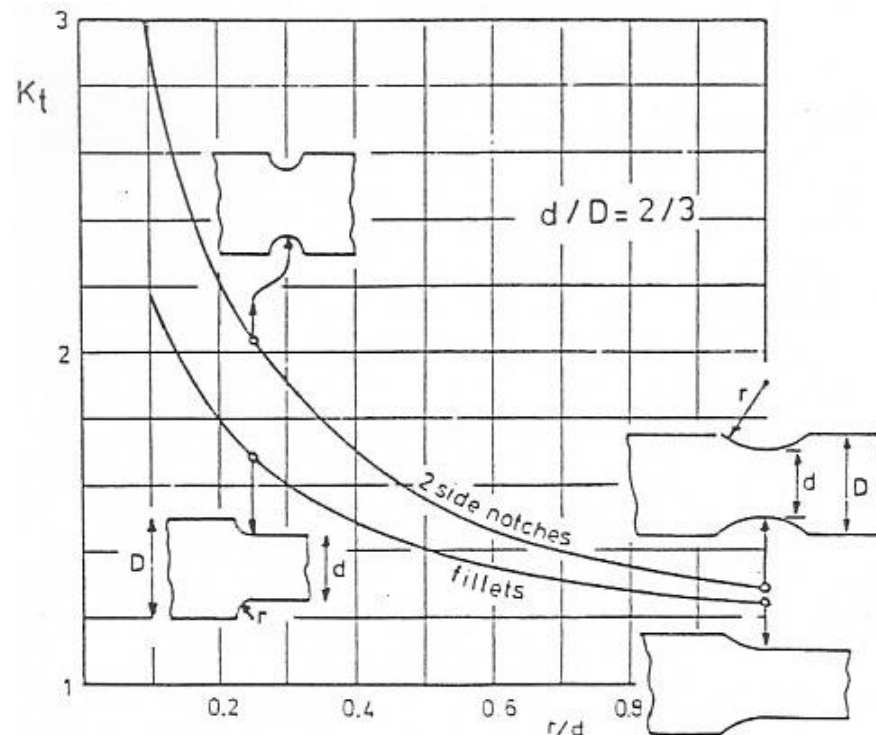
- Tensile load: $P = 24 \text{ kN}$
Sheet thickness: $t = 5 \text{ mm}$

$$\sigma_{\max} = K_t \frac{P}{(W - 2a) \cdot t} = 3.46 \frac{24000}{(40 - 11.6) \cdot 5} = 585 \text{ MPa}$$

Exercise

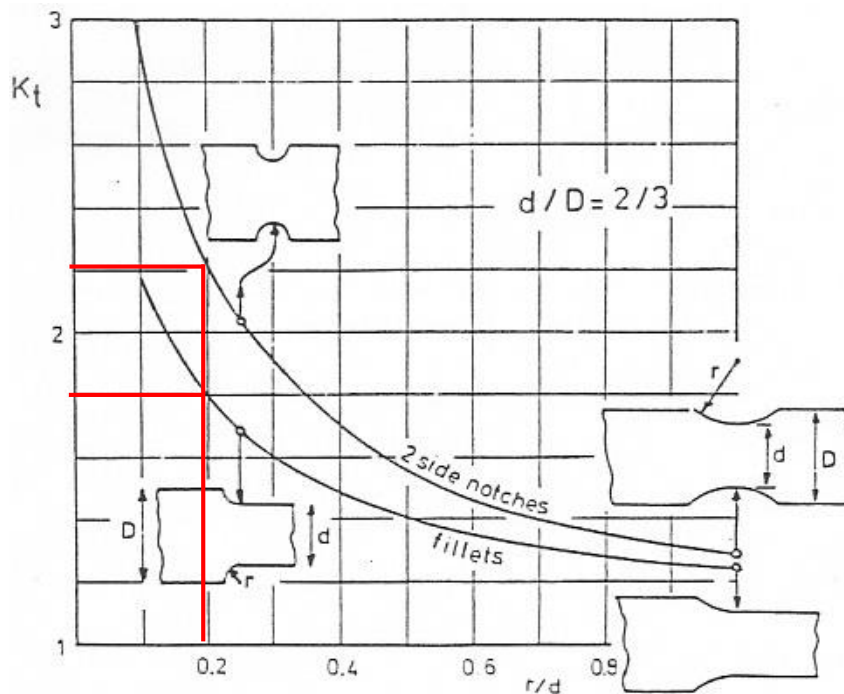
Discuss using the diagram below what is better for fatigue: Letters forged on the surface of a component or letters forged in the surface.

The letters can be considered as either a protruding fillet or a rounded groove with $r/d = 0.2$.



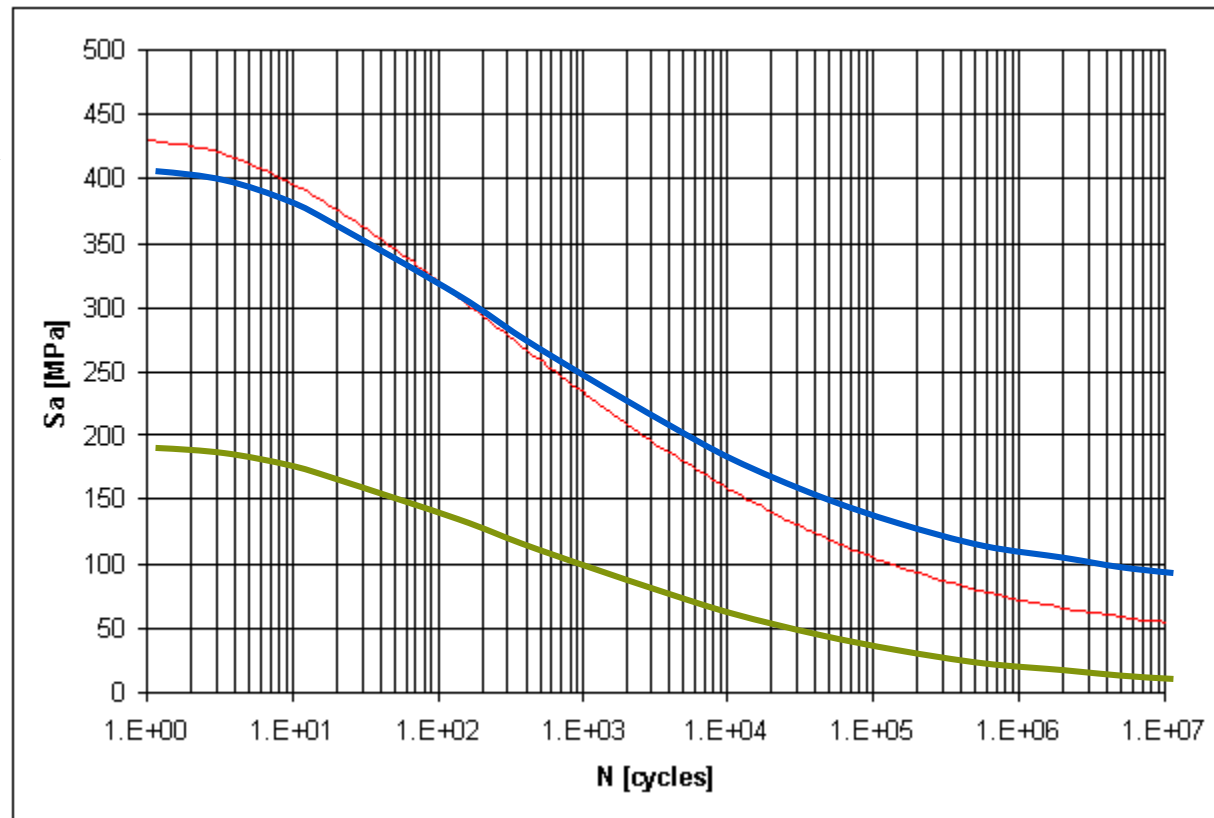
Exercise

- Letters forged on the surface: $K_t = 1.8$
- Letters forged in the surface: $K_t = 2.2$
- Letters forged on the surface lead to lower peak stresses and therefore a better fatigue life.



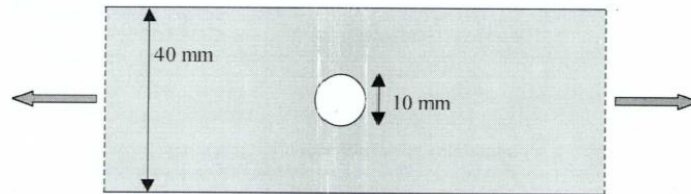
Exercise

- An S-N curve is given for 2024-T351, $K_t = 2.5$, $S_m = 0$ MPa. Make a sketch that indicates where the S-N curve would lie compared to the curve in the figure below for the following two cases:
 - 2024-T351, $K_t = 1$,
 $S_m = 0$ MPa
 - 2024-T351, $K_t = 2.5$,
 $R = 0$.



Exercise

- Estimate the fatigue limit for the strip with a central hole ($K_t = 2.42$) as shown in the figure. Use the Neuber notch sensitivity (why not Siebel?). The material is Al 7075-T6, $R_m = 570$ MPa.
 - a) Fatigue limit for $R=-1$ ($S_m = 0$ MPa)
 - b) Fatigue limit for $R=0$ using the Goodman relation



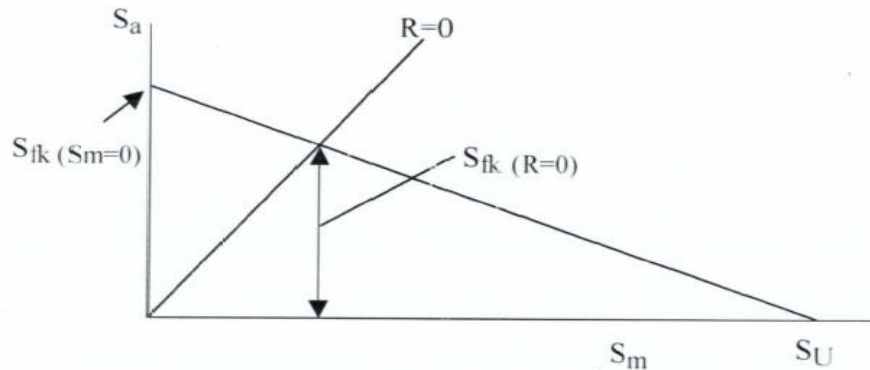
- $S_f = 0.3 \cdot 570 = 171$ MPa for $R=-1$, un-notched (for aluminium: $S_f/R_m \approx 0.3$)
- Neuber notch sensitivity:

$$K_f = 1 + q \cdot (K_t - 1) \quad q_{Neuber} = \frac{1}{1 + \sqrt{A/\rho}}$$

- $A = 0.5$; $\rho = D/2 = 5$; $q = 0.76 \Rightarrow K_f = 2.08$
- $S_{fk(S_m=0)} = S_{f(S_m=0)} / K_f = 171 / 2.08 = 82$ MPa

Exercise

- Because S_{fk} should be obtained for $R = 0$, the effect of the mean stress S_m should be accounted for. This is with the Goodman relation (brittle material).



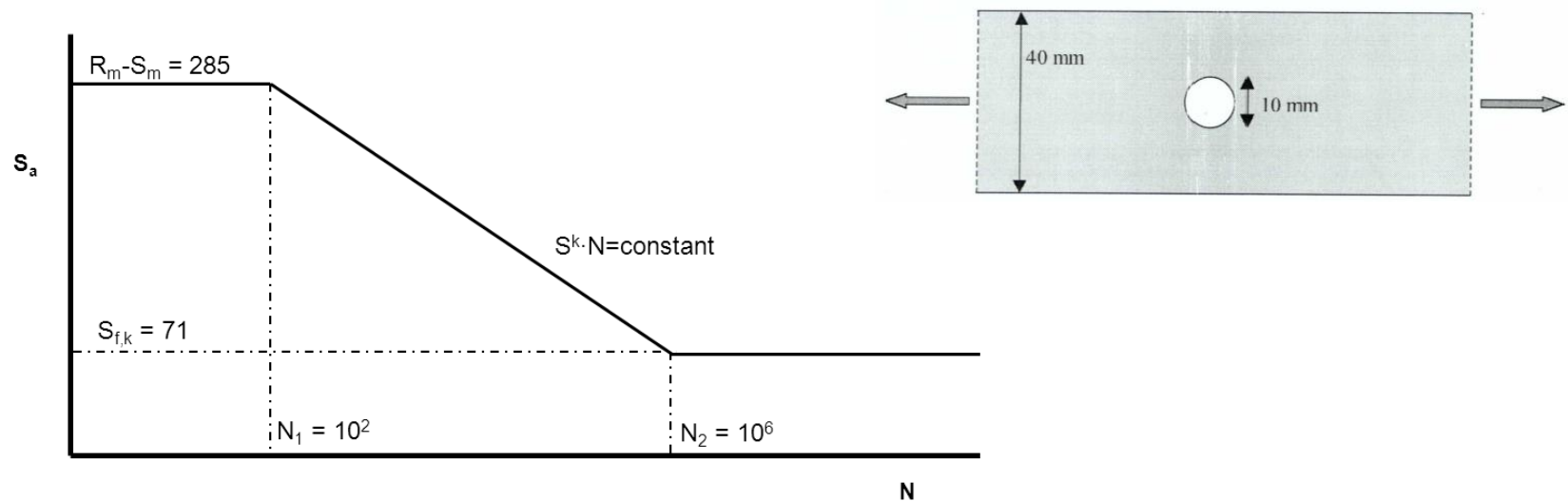
- Goodman relation:

$$\frac{S_f}{(S_f)_{S_m=0}} = 1 - \left(\frac{S_m}{R_m} \right)$$

- $R=0$: $S_m = S_a$
- Intersection of lines for $R=0$ and Goodman gives:
- $S_{fk(R=-1)} = 82 \text{ MPa}$ & $R_m = 570 \text{ MPa} \Rightarrow S_{fk(R=0)} = 71 \text{ MPa}$

Exercise

- Estimate the S-N curve for R=0 for the strip with a central hole ($K_t = 2.42$) as shown in the figure. The material is Al 7075-T6, $R_m = 570$ MPa.
 - From previous exercise: $S_{fk(R=0)} = 71$ MPa



- Upper asymptote: $R_m = S_{\max} = S_m + S_a \Rightarrow S_a = R_m - S_m$
 For R=0: $S_a = S_m \Rightarrow S_a = R_m - S_a \Rightarrow S_a = R_m/2 = 570/2 = 285$ MPa
- Lower asymptote: $S_{fk} = 71$ MPa